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by Xiaogang Huang, Maureen E. Foley, Travis A. Bogetti, and John W. Gillespie, Jr.

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# **Mechanics of the Fiber-Matrix Interphase Push-Out Test**

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#### Abstract

The stress distribution for the fiber-matrix interphase push-out test is derived. The analytical solution provides the mechanics foundation to relate experimental measurements to the properties of the interphase. The shear-lag model incorporates a discrete interphase region quantified by the thickness and shear modulus of the interphase. Except for the basic shear-lag model assumptions, the method follows a rigorous mathematical approach. Accuracy of the analytical solution is assessed using finite element analysis. The data reduction scheme enables the interphase strength and shear modulus to be determined from load-deformation response of the specimen.

# 1. Introduction

The material immediately surrounding the fiber can be significantly different from the bulk matrix [1–4]. This thin-layer of material is called the fiber-matrix interphase. The properties and microstructure of the interphase material govern the load transfer between the composite constituents. The fiber-matrix interphase has significant influence on the structural integrity of fibrous composites. This has led to numerous efforts in both the experimental characterization and micro-mechanical analysis of the interphase subjected to different loading conditions.

The traditional methods for predicting the stress distribution in the fiber matrix region are shear-lag theory [5–8] and finite element analysis (FEA) [9–11]. Most studies neglect the interphase because the properties and dimensions are unknown. Hence the presence of the interphase is commonly replaced by an interface between the fiber and the matrix. Recently, experimental efforts [12&13] have been conducted to characterize the interphase properties and its effects on the macroscopic properties of the composite.

The aim of the present work is to develop an analytical based mechanics solution that can be used as the data reduction procedure for the fiber push-out experiment to evaluate the fiber-matrix interphase properties. Based on shear-lag theory, the interphase is assumed to be a uniformly thin layer surrounding the fiber having constant stiffness. The interphase shear strain was derived as a function of the experimentally measured quantities (i.e. applied load and measured displacement) from the push-out test. In

addition to the analytical solution, a finite element model (FEM) for the push-out test is developed and compared to the theoretical predictions.

# 2. Theory

In this section, an analytical model is presented. Based on the governing equations and the shear-lag assumptions, the displacements in each region of the domains are derived. In the specimen, there are three axially symmetric regions (as shown in Figure 1): fiber, matrix and the interphase between the fiber and the matrix.

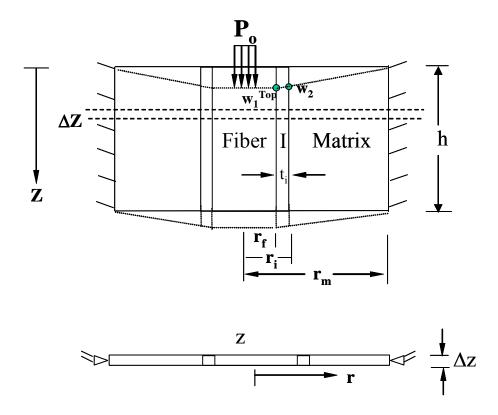


Figure 1 Specimen geometry and boundary conditions for the fiber push-out test

A single fiber composite of thickness h, is surrounded by an interphase of thickness  $t_i$  and supported by an annulus of radius  $r_m$ . A schematic diagram of the unloaded and loaded cases of this fiber push-out configuration is given in Figure 1. The displacement  $w_1^{\text{Top}}$  and distance z are taken from the top surface. It is assumed that the fiber is rigid, carries only axial load and has a displacement of  $w_1$ . The matrix and interphase are assumed to undergo shear deformation only. It is assumed that the fiber, matrix, and interphase are perfectly bonded together. Consequently, no relative sliding

between the materials is allowed. The displacement of the interphase at  $r = r_i$  is defined as  $w_2$ . Outside the matrix cylinder,  $r=r_m$ , the boundary is assumed fixed.

The force balance equation can be presented in fiber and matrix regions as,

$$\tau r = \tau_i r_i = \tau_m r_m, \quad \tau = \frac{\tau_i r_i}{r} = \frac{\tau_m r_m}{r}$$

$$\tau_i = \frac{P_o}{2\pi h r_i}, \quad \tau_m = \frac{P_o}{2\pi h r_m}$$
(1)

where the r stands for radius, P<sub>o</sub> is the applied force, h is the specimen height and subscripts f, i and m stand for fiber, interphase and matrix, respectively.

The basic governing differential equation is given by:

$$dw = \frac{\tau}{G} dr \tag{2}$$

In the interphase and matrix regions, the displacements are solved from equation 2 as,

$$r_{f} < r < r_{i}$$
:

$$w_i = \frac{\tau_i}{G_i} r_i \ln r + C_1 \tag{3}$$

 $r_i < r < r_m$ :

$$W_{m} = \frac{\tau_{m}}{G_{m}} r_{m} \ln r + C_{2} \tag{4}$$

By applying the boundary condition  $w_m = 0$  at  $r = r_m$ , the constant  $C_2$  can be obtained:

$$C_2 = -\frac{\tau_m}{G_m} r_m \ln r_m \tag{5}$$

Substitution of Equation 5 into Equation 4 gives the expression for deflection in the matrix:

$$W_{m} = \frac{\tau_{m}}{G_{m}} r_{m} \ln \frac{r}{r_{m}}$$
 (6)

Along the interface between the interphase and matrix (axial direction), the displacement is continuous (i.e.  $w_i = w_m$  at  $r = r_i$ ). Using this condition allows one to solve for  $C_1$ . The deflection in the interphase is:

$$w_{i} = \frac{\tau_{i}}{G_{i}} r_{i} \ln \frac{r}{r_{i}} + \frac{\tau_{m}}{G_{m}} r_{m} \ln \frac{r_{i}}{r_{m}}$$
(7)

For data reduction purposes, the interphase shear stain  $\gamma_i$  needs to be expressed in terms of the load  $(P_o)$  and displacement applied to the top of the fiber  $(w_1(z=0))$ . This deflection is designated  $w_1^{Top}$  in Figure 1. From Equation 7, and by considering  $w_i = w_1$  at  $r = r_f$ , the interface displacement and the general form of interphase shear strain are given by:

$$w_1 = \gamma_i r_i \ln \frac{r_f}{r_i} + \frac{\tau_m}{G_m} r_m \ln \frac{r_i}{r_m}$$
(8)

$$\gamma_{i} = w_{l} \left( \frac{\frac{\tau_{m}/w_{l}}{G_{m}} r_{m} \ln \frac{r_{i}}{r_{m}} - 1}{r_{i} \ln \frac{r_{i}}{r_{f}}} \right)$$

$$(9)$$

However,  $w_1$  and  $\gamma_i$  vary from point to point along the axial direction. It is necessary to consider the variation in the axial (z) direction. The axial stress is a maximum at the point of load introduction and is equal to  $P_o/\pi r_f^2$ . By definition,

$$\varepsilon = \frac{\mathrm{dw}_1}{\mathrm{dz}} \tag{10}$$

Therefore, since,  $A_f = \pi r_f^2$ , we have,

$$F = E_f \pi r_f^2 \frac{dw_1}{dz} \tag{11}$$

Force Balance yields the following differential equations for the interphase and matrix:

$$\begin{split} \frac{dF}{dz} + 2\pi r_i \tau_i &= 0 \\ \frac{dF}{dz} + 2\pi r_m \tau_m &= 0 \end{split} \tag{12}$$

Substituting into Equation 11 yields:

$$\begin{split} E_{f}\pi r_{f}^{2} \frac{d^{2}w_{1}}{dz^{2}} + 2\pi r_{i}\tau_{i} &= 0\\ E_{f}\pi r_{f}^{2} \frac{d^{2}w_{1}}{dz^{2}} + 2\pi r_{m}\tau_{m} &= 0 \end{split} \tag{13}$$

Substituting Equations 8 and 9 into Equation 13 yields:

$$\frac{d^{2}w_{1}}{dz^{2}} - \frac{2G_{i}}{E_{f}r_{f}^{2} \ln \binom{r_{i}}{r_{f}}} (w_{1} - w_{2}) = 0$$

$$\frac{d^{2}w_{1}}{dz^{2}} - \frac{2G_{m}}{E_{f}r_{f}^{2} \ln \binom{r_{m}}{r_{f}}} w_{2} = 0$$
(14)

For simplicity, we manipulate Equation 14 into the form of

$$\frac{d^2 w_1}{dz^2} - a^2 w_1 = 0 ag{15}$$

where

$$a^{2} = \frac{2G_{i}/(E_{f}r_{f}^{2}\ln(r_{i}/r_{f}))}{1 - \frac{G_{i}\ln(r_{i}/r_{m})}{G_{m}\ln(r_{i}/r_{f})}}$$

By solving Equation 15, we have

$$w_1 = C_4 \cosh(az) + C_5 \sinh(az)$$
 (16)

The boundary conditions along the axial direction are,

$$z = 0, P = -P_o = E_f \pi r_f^2 \frac{dw_1}{dz}$$

$$z = h, P = 0, \frac{dw_1}{dz} = 0$$
(17)

Combining Equations 16 and 17 and solving for the unknown coefficients yields the generalized expression for  $w_1$ :

$$w_1(z) = \frac{-P_o}{r_f^2 E_f a \pi} \left[ -\coth(ah)\cosh(az) + \sinh(az) \right]$$
 (18)

The solution given by Equation 18 is the displacement of the interface between the fiber and the interphase (i.e.  $r=r_f$ ).

For the special case z=0, Equation 18 becomes the top displacement at the fiber-interphase interface,

$$w_1(z=0) = w_1^{Top} = \frac{P_o}{r_f^2 E_f a \pi} \coth(ah)$$
 (19)

In the fiber push-out test, the  $w_1^{Top}$  is measured directly using a displacement sensor and is therefore a useful parameter in the data reduction procedure. By considering equation 9 and after some mathematical manipulation, the interphase shear strain can be explicitly expressed in terms of  $w_1(z)$  and  $w_1^{Top}$  as,

$$\gamma_{i}(z) = \frac{w_{1}^{Top} \left[\frac{-B}{1-B}\right]}{\frac{-P_{o}}{r_{f}^{2}E_{f}a\pi} \coth(ah)r_{i}\ln(r_{i}/r_{f})} w_{1}(z)$$

$$B = \frac{G_{i}}{G_{m}} \frac{\ln \frac{r_{i}}{r_{m}}}{\ln \frac{r_{i}}{r_{f}}}$$
(20)

Equation 20 shows that the shear strain is directly proportional to the amplitude of the applied displacement. In addition, the distribution of the shear strain in the axial direction follows the function  $w_1(z)$ . This function is presented in Equation 18 where  $w_1$  at the location of load introduction is much higher than the value at the bottom of the sample. For materials characterization this gradient can be minimized by specimen design but can not be eliminated. An average displacement  $w_1$  in the axial direction can be defined in an integral form as

$$\overline{\mathbf{w}}_{1}(\mathbf{z}) = \frac{1}{h} \int_{0}^{h} \mathbf{w}_{1}(\mathbf{z}) \tag{21}$$

where h is the test sample thickness.

For data reduction purposes, an average shear strain is derived to define the effective interphase shear modulus. This interphase shear modulus is defined as the

proportionality constant relating average shear stress (Equation 1) to the average shear strain in the interphase. By definition of average value and combining Equation 20 and 21, we express the average interphase shear strain in integral form as,

$$\begin{split} \bar{\gamma}_{i} &= \frac{w_{1}^{Top} \left[ \frac{-B}{1-B} - 1 \right]}{\frac{-P_{o}}{r_{f}^{2} E_{f} a \pi} \coth(ah) r_{i} \ln(r_{i}/r_{f})} \overline{w}_{1}(z) = \frac{w_{1}^{Top} \left[ \frac{-B}{1-B} - 1 \right]}{\frac{-P_{o}}{r_{f}^{2} E_{f} a \pi} \coth(ah) r_{i} \ln(r_{i}/r_{f})} \bullet \frac{1}{h} \int_{o}^{h} w_{1}(z) dz \\ &= \frac{w_{1}^{Top} \left[ \frac{-B}{1-B} - 1 \right]}{\coth(ah) r_{i} \ln(r_{i}/r_{f})} \bullet \frac{1}{h} \int_{o}^{h} \left[ -\coth(ah) \cosh(az) + \sinh(az) \right] dz \\ &= \frac{w_{1}^{Top}}{\coth(ah) r_{i} \ln(r_{i}/r_{f})} \bullet \frac{1}{ah(1-B)} \end{split}$$
 (22)

Finally, we can obtain the interphase shear modulus in terms of average shear strain and the experimentally measured parameters of applied load and displacement.

$$G_{i} = \frac{\tau_{i}}{\overline{\gamma}_{i}} = \left(\frac{P_{o}}{w_{1}^{Top}}\right) \frac{\coth(ah)ar_{i}\ln(r_{i}/r_{f})(1-B)}{2r_{f}\pi}$$
(23)

It should be mentioned that because the parameter a (in Equation 15) contains the interphase shear modulus  $G_i$ , the Equation 23 is implicit and requires an iterative solution. The ratio of load  $P_o$  vs. the top displacement  $w_1^{Top}$  represents the linear slope of the loading-displacement diagram recorded from the test. In the experimental data reduction, the specimen geometric size and the material properties of fiber and matrix are assumed to be known constants. An assumed initial value is needed for the interphase shear modulus  $G_i$  to iterate and converge to a unique value.

# 3. Finite Element Approach

As a verification of the closed form data reduction analysis, a finite element model (FEM) of the fiber push-out test is developed to correlate with the analytical approach. The FEM addresses all the concerns noted in the analytical solution. The geometry of the fiber push-out model is the same as in Figure 1. An axisymmetric model is defined with r as the radial coordinate with origin at the fiber central axis and z as the axial coordinate origin at the top of the model. The radii of the fiber, interphase, and out edge of the matrix are  $r_{\rm f}$ ,  $r_{\rm i}$ , and  $r_{\rm m}$ , respectively. The model thickness in the axial direction is denoted by h.

The commercially available finite element code ABAOUS (Hibbitt, Karlsson, and Sorenson, Inc.) [14] was used for FEM analysis. The fiber push-out model was modeled with a first-order axisymmetric isoparametric element. The fiber and interphase and matrix materials were assumed to be isotropic and linear elastic. The top and bottom regions of the model were densely meshed because of the stress singularity near the free edges. Because the interphase thickness is very small compared with the fiber radius, the total number of elements in the model is relatively large (30,000 elements were used.). In the simulation, a known interphase modulus is specified. The push-out load,  $P_o$ , was applied and the top displacement,  $w_1^{Top}$ , was generated from the numerical model. Using these values in the data reduction scheme for P<sub>o</sub>/w<sub>1</sub><sup>Top</sup> in Equation 23, the resulting interphase shear modulus is calculated and should be equal to the interphase modulus G<sub>i</sub> used in the FEM. Deviations from the known properties is our metric to evaluate the accuracy of the data reduction procedure for this numerical experiment. The Glass fiber/Epoxy and Glass fiber/Vinyl ester matrix composites were used in the correlation studies. The interphase displacement, shear strain and shear stress are examined and compared for both approaches. The details of the fiber-matrix property used in the model are listed in Table 1.

Property	Glass Fiber/Epoxy	Glass Fiber/Vinyl Ester	
Elastic Modulus – Fiber	72.6 GPa	72.6 GPa	
Elastic Modulus – Matrix	2.4 GPa	3.4 GPa	
Elastic Modulus - Interphase	246 MPa	250 MPa	
Fiber Diameter, 2r <sub>f</sub>	20 μm	17 μm	
Matrix Radius r <sub>m</sub>	25 μm	25 μm	
Interphase Thickness, (r <sub>i</sub> -r <sub>f</sub> )	0.07 μm	0.12 μm	
Sample Thickness, h	83 μm	150 μm	

Table 1. Composite Material Properties

From the analytical derivation and Equation 18, it is seen that the displacement  $w_1$  is a critical parameter in the data reduction. Since  $w_1$  varies from place to place along the z direction it's impossible to measure every point.  $W_1(z=0)$  is a measurable value in the experiment, while an average  $w_1(z)$  is directly related to interphase shear strain  $\gamma_i$ . The FEM simulation for the  $w_1$  is conducted and compared with the analytical solution (Equation 18). The glass fiber/epoxy matrix composite was used in this comparison. Figure 2 shows the distributions for  $w_1(z)$  as the function of the axial location z from both analytical and FEM results. It is noted that the top displacement is more than double the displacement at the bottom of the specimen in this example. Although the analytical solution cannot capture the values near the top interface area due to free edge effects, the interior displacement values are very close to the FEA result.

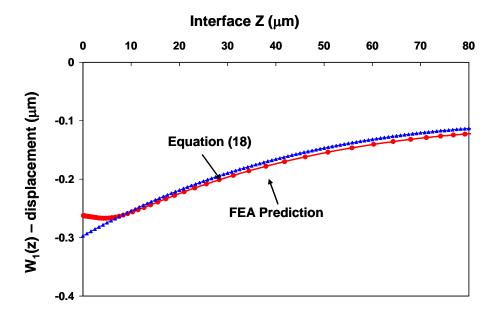


Figure 2. The displacement  $w_1$  distribution for Glass fiber/Epoxy composite along the interface

The interphase shear strain dominants the interphase deformation. Due to the free edge singularity effect, the shear strain also varies dramatically near the bottom and top edges. The shear strain distributions along the interface from the FEA and analytical solution Equation (20) are plotted in Figure 3. It is seen that except for the free edges the analytical solution fits the FEA prediction very well. In terms of average value, the analytical strain is 2.5% higher than the averaged FEA results.

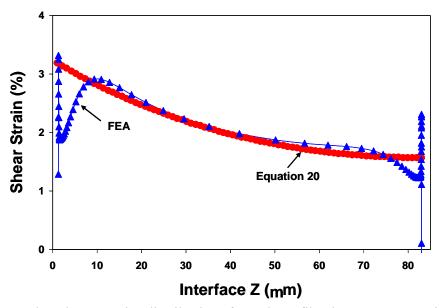


Figure 3. The shear strain distribution for Glass fiber/Epoxy composite along the interface

Figure 4 shows the interphase shear stress along the interface axial direction variations for FEA and analytical solution. The shear stress distribution is in good agreement in the interior and deviates at the free edges. The average shear stress of the distribution is equal to the applied load divided by the shear area. For this specimen geometry, the shear stress varies by a factor of two over the specimen thickness.

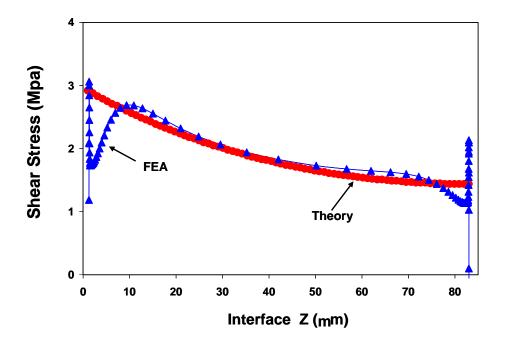


Figure 4. The shear stress distribution for Glass fiber/Epoxy composite along the interface

# 4. Parametric Study, Comparison and Discussion

A wide range of interphase shear moduli were considered in this parametric study (0.96 MPa to 246 MPa). Table 2 summarizes the results for the analytical and numerical models. One observes that the errors are reduced as the interphase becomes less stiff (i.e.  $G_m/G_i$  increases). The maximum error for interphase shear modulus is <2% in the case of high interphase stiffness. The range of moduli considered is quite realistic for the glass/epoxy composite. Using material properties and geometry in Table 1 and published load-displacement diagram from [12], the average interphase shear modulus obtained from Equation 23 was 25 MPa. Based on the results presented in Table 2, the data reduction should be accurate within 2%.

	Analytical Model		FEA		G. <sup>Anal</sup>		
	τ (Pa)	γ	G <sub>i</sub> (MPa)	τ (Pa)	γ	G <sub>i</sub> (MPa)	$\frac{G_{i}^{1}}{G_{i}^{FEA}}*100\%$
$G_{\rm m}/G_{\rm i} = 3.9$ $G_{\rm i} = 245 \text{ MPa}$	159	6.355e-7	250	159	6.346e-7	246	102 %
$G_{\rm m}/G_{\rm i} = 10$ $G_{\rm i} = 96.2 \text{ MPa}$	159	1.630e-6	96.3	159	1.65e-6	96.4	101 %
$G_{\rm m}/G_{\rm i} = 100$ $G_{\rm i} = 9.62 \text{ MPa}$	159	1.651e-5	9.63	159	1.65e-5	9.64	100 %
$G_{\rm m}/G_{\rm i} = 1000$ $G_{\rm i} = 0.962 \text{ MPa}$	159	1.652e-4	0.963	159	1.65e-4	0.964	100 %

Table 2. Accuracy of the Analytic Solution versus FEA for a wide range of interphase moduli

The analytical model equations can be studied further to determine the effect of varying certain parameters on the calculated values. The variability of the interphase modulus,  $G_i$  will be investigated as a function of interphase thickness and the slope of the load-displacement curve. One of the important inputs into the analytical model is the interphase thickness value. This had been estimated in the past through swelling theory or estimation of sizing thickness present on the fiber. Equation 23 was used to determine the effect of interphase thickness on the calculated  $G_m/G_i$  value for a given system as a function of  $P_o/w_1^{Top}$ . The parametric study was performed using a glass fiber reinforced vinyl ester composite system, shown in Table 1, with an  $r_m$ =32.5 $\mu$ m. An iterative program was written to determine the  $G_m/G_i$  values over a range of  $P_o/w_1^{Top}$ . The program is necessary due to the interdependence of the equation on  $G_m/G_i$ . It is first necessary to choose a  $G_m/G_i$  value, perform the calculations, compare and then chose a new value and iterate until the values match. The results of these calculations are shown graphically in Figure 5.

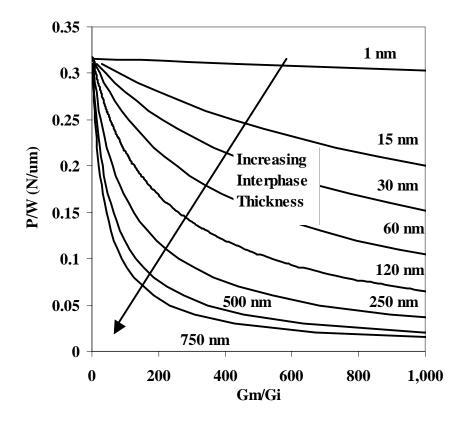


Figure 5. Effect of Interphase Thickness on  $G_m/G_i$  for a given  $P_o/W_1^{Top}$  (Glass Fiber Reinforced Vinyl Ester System).

The figure highlights the importance of the correct estimation of the interphase thickness. As the interphase thickness increases, the  $G_m/G_i$  value decreases dramatically for a given  $P_o/{w_1}^{Top}$  value. Typically the interphase modulus is not greater than the matrix modulus therefore the minimum values in Figure 5 are plotted are for  $G_m/G_i \geq 1$ . The maximum value of  $G_m/G_i$  =1000 was arbitrarily chosen as an approximate  $G_m/G_i$  ratio for rubber-like properties of the interphase. Alternatively, if one considers the low range of  $G_m/G_i$  values, one finds that the stiffness  $(P_o/{w_1}^{Top})$  increases dramatically as the interphase thickness decreases.

Experimentally, the value of  $P_o/w_1^{Top}$  typically has a variability of about  $\pm 0.01$  N/µm. This amount of variation would not have much effect on the  $G_m/G_i$  value of large interphases, but would have a significant effect on those with small interphases. For example, for a sample that exhibited a  $P_o/W_1^{Top}$  of  $0.1\pm0.01$ , this would lead to a  $G_m/G_i$  range of 2521 to 1832 for an interphase thickness of 30 nm and a range of 106 to 77 for an interphase thickness of 750 nm. Assuming a matrix modulus of 1.31 GPa, this converts to an interphase modulus range of 0.52 to 0.71 MPa for the 30 nm interphase thickness and a range of 12.3 to 16.9 MPa for an interphase thickness of 750 nm.

#### 5. Conclusions

An implicit analytical solution was developed to characterize the fiber-matrix interphase mechanical properties for the fiber-matrix push-out test. The interphase shear modulus can be expressed in terms of known fiber-matrix properties and push-out test measurable load-deflection curve  $P_o/w_1^{Top}$ . The FEA model was developed for correlation study with the analytical solution. The glass fiber epoxy matrix composite was studied as an example for comparison and the glass fiber vinyl ester matrix composite was used for parametric study. The satisfactory comparison results for the interphase displacement, shear strain and shear stress were obtained. Given typically available experiment data such as  $P_o/w_1^{Top}$ , the interphase shear modulus can be uniquely determined. A parametric study, using the analytical model, showed that the interphase thickness value has a significant effect on the calculated interphase shear modulus. More significantly, the approach can be extended to the strain rate dependent interphase characterization of the fiber-matrix push-out test, which is partially conducted in references [12] and [13].

#### 6. References

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